V1

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$Q(x,y,z) = x^2 + y^2 + z^2 + xy + yz + zx$

Esitysmatriisi$(x\,y\,z)$ $\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$= (ax + by + zc)x + (xb + yd + ze)y + (xc + ye + zf)z$

$= ax^2 + byx + zcx + xby + dy^2 + zey + xcz + yez + fz^2$

$= ax^2 + by^2 + fz^2 + 2byx + 2czx + 2eyz$

$A = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

$Ax = \lambda x$ Määritetään $C = A - \lambda I$

$(A - \lambda I)x = 0$

$C = \frac{1}{2}\begin{pmatrix} 2-2\lambda & 1 & 1 \\ 1 & 2-2\lambda & 1 \\ 1 & 1 & 2-2\lambda \end{pmatrix}$

$\det C = -\lambda^3 + 3\lambda^2 - \frac{9}{4}\lambda + \frac{1}{2}$ kaikki positiivisia

$\lambda_1 = \frac{1}{2}$ $\lambda_2 = \frac{1}{2}$ $\lambda_3 = 2$ eli positiivisesti definiitti

$\lambda_2 = 9$

$$\begin{array}{cc|c} -4 & 2 & 0 \\ 2 & -1 & 0 \end{array} \downarrow \cdot \tfrac{1}{2}$$

$$\begin{array}{cc|c} -4 & 2 & 0 \\ 0 & 0 & 0 \end{array} \quad || : 2$$

$$\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & 0 & 0 \end{array}$$

$x_2 = \beta$ | 1 tukialkio vapaa

$-2x + \beta = 0$

$x_1 = \frac{1}{2}\beta$

$$V_2 = \begin{pmatrix} \frac{1}{2}\beta \\ \beta \end{pmatrix}$$

Olkoon $\beta = 2$

$V_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, ortonormitettu $V_{2n} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}^T$

ortonormitettu ominaisvektorit V_{1n} ja V_{2n} muodostavat similariteetti-muunnosmatriisi T:n

$$T = V_2 \; V_1 = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$\Lambda = \begin{matrix} 9 & 0 \\ 0 & 4 \end{matrix}$$

$$T^{-1} = T^T = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$T^{-1}AT = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 2 & 8 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix} = \Lambda$$

V2

a) $A = \begin{pmatrix} 5 & 2 \\ 2 & 8 \end{pmatrix}$

$A - \lambda I = \begin{pmatrix} 5-\lambda & 2 \\ 2 & 8-\lambda \end{pmatrix}$

$\det(A - \lambda I) = (5-\lambda)(8-\lambda) - 4$

$\det(A - \lambda I) = 0$

$(5-\lambda)(8-\lambda) - 4 = 0$

$\lambda_1 = 4$

$\lambda_2 = 9$

määritetään ominaisvektorit:

$\lambda_1 = 4$

$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 4 & 0 \end{array}\right) \downarrow -2$

$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$x_1 = \alpha$

$x_1 + 2\alpha = 0$

Olkoon $\alpha = 1$

$v_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

lasketaan v_1 suuntainen yksikkövektori

$v_{1n} = \frac{1}{\sqrt{5}} (-2 \;\; 1)^T$